The Robot Doctor

Episode 108: Robot Controls

Common Core Standards:

- Vectors:
  - Vector Format of a Line using the start and end points of the line (P_{end} - P_{start})
  - Cross Product of 2D Vectors
  - Using Dot Product and Normal Vector of a line to calculate the distance from a point to the line
- Pythagorean Theorem

Review:

The trajectory is the path the robot is supposed to be following. It is made up of waypoints that have position, and maybe velocity or time or other variables associated with each of them.

Proportional Feedback Control is a common solution for keeping a robot on a trajectory.

We'll use the vector notation for a line and define two in particular:

$$\overrightarrow{line}_{start\ to\ end} = \overrightarrow{line}_{S\to E} = \begin{bmatrix} End_x - Start_x \\ End_y - Start_y \end{bmatrix}$$

$$\overrightarrow{line}_{start\ to\ robot} = \overrightarrow{line}_{S\to R} = \begin{bmatrix} robot_x - start_x \\ robot_y - start_y \end{bmatrix}$$

We can use the cross product to determine which side we are on:

$$side = \overrightarrow{line}_{S\to E} \times \overrightarrow{line}_{S\to R} = \begin{bmatrix} 0 \\ 10 \end{bmatrix} \times \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} A \\ C \end{bmatrix} \times \begin{bmatrix} B \\ D \end{bmatrix} \to AD - BC$$

Review:

Negative means we are on the right side, positive on the left side.

We can find our distance from the line using the dot product and the normal vector:

$$distance = \frac{\left|\overrightarrow{line_{S \to R}} \cdot \overrightarrow{normal}\right|}{\left\|\overrightarrow{normal}\right\|}$$

We can find the normal by taking the original line, swapping the x and y coordinates, and negating one of them.

$$\overrightarrow{Line} = \begin{bmatrix} X \\ Y \end{bmatrix} \to \overrightarrow{Normal} = \begin{bmatrix} -Y \\ X \end{bmatrix}$$

For the magnitude, we take the Pythagorean theorem to find the total length:

$$a^2 + b^2 = c^2$$

Where a and b are the x and y components, and c is the total length.

$$\begin{bmatrix} A \\ C \end{bmatrix} \cdot \begin{bmatrix} B \\ D \end{bmatrix} \to AB + CD$$

For the dot product, we multiply the x terms of both arguments, multiply the two y terms, then add the results.

Challenge Questions

Our robot wants to follow a diagonal line going through the origin and a point at 10,10.

Question 1 – if the robot is at 2,3, how far away from the line is the robot? And on which side?

Question 2 – if the proportional gain is 15 degrees per meter, what is the commanded steering angle from the controller?